

◇ Useful formulas

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0(1 + \chi_e) \mathbf{E} = \epsilon \mathbf{E} \quad \text{and} \quad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi_m) \mathbf{H} = \mu \mathbf{H}$$

$$\nabla \cdot \mathbf{D} = \rho_f \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad \nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_f$$

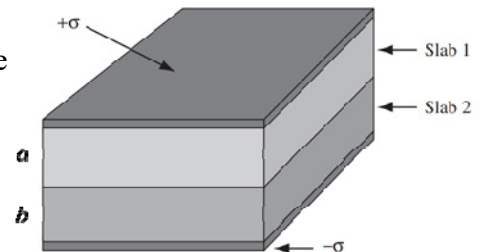
$$D_1^\perp - D_2^\perp = \sigma_f \quad \mathbf{E}_1'' - \mathbf{E}_2'' = 0 \quad B_1^\perp - B_2^\perp = 0 \quad \mathbf{H}_1'' - \mathbf{H}_2'' = (\mathbf{K}_f \times \hat{\mathbf{n}})$$

1. Explain the following terms. Write down the mathematical expressions *if applicable*. (6×5%)

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|---|-------------------------------------|
| (a) Electric susceptibility (5%) | (c) The Coulomb gauge (5%) |
| (b) Ampere's law (5%) | (d) Ohm's law with Drude model (5%) |
| (e) Hysteresis loop and Curie temperature (5%) | |
| (f) Effect of a magnetic field on atomic orbits and diamagnetism (5%) | |

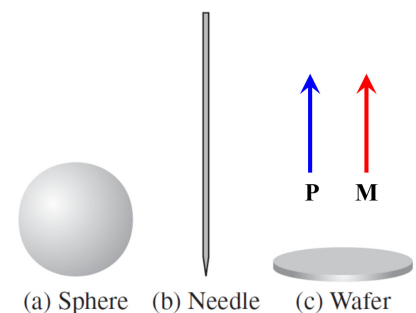
2. The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. The thicknesses of the slabs 1 and 2 are a and b respectively. Slab 1 has a dielectric constant of 9 and slab 2 has a dielectric constant of 4. The free charge density of the top plate is σ and on the bottom plate $-\sigma$.

- (a) Find the electric displacement $\mathbf{D}$, the electric field $\mathbf{E}$, and the polarization $\mathbf{P}$ in each slab. (5%)
 (b) Find the location and amount of all bound charges (ρ_b and σ_b). (5%)
 (c) Find the potential difference V between the metal plates. (5%)
 (d) Find the capacitance of the capacitor. (5%) [Hint: Consider the parallel-plate capacitor with the surface area A .]



2. Suppose the fields inside a large piece of material are $\mathbf{E}_0$ and $\mathbf{B}_0$, so that $\mathbf{D}_0 = \epsilon_0 \mathbf{E}_0 + \mathbf{P}$ and $\mathbf{B}_0 = \mu_0(\mathbf{H}_0 + \mathbf{M})$, where $\mathbf{P}$ and $\mathbf{M}$ are “frozen-in”, so they don't change when the cavity is excavated.

- (a) Now a small spherical cavity (Fig. (a)) is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$. Also find the displacement at the center of the cavity in terms of $\mathbf{D}_0$ and $\mathbf{P}$. (5%)
 (b) Do the same for a long needle-shaped cavity (Fig. (b)) running parallel to $\mathbf{P}$. (5%)



(c) Now a small spherical cavity (Fig. (a)) is hollowed out of the material. Find the field at the center of the cavity, in terms of $\mathbf{B}_0$ and $\mathbf{M}$. Also find $\mathbf{H}$ at the center of the cavity, in terms of $\mathbf{H}_0$ and $\mathbf{M}$. (5%)

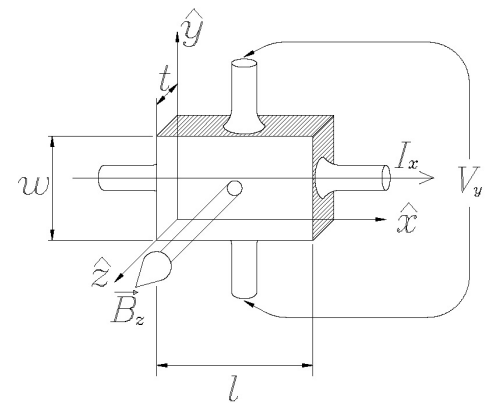
(d) Do the same for a thin wafer-shaped cavity (Fig. (c)) perpendicular to $\mathbf{M}$. [Hint: Assume the cavities are small enough. Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.] (5%)

4. (7%,7%,6%) Consider a conducting slab as shown below with length l in the x direction, width w in the y direction and thickness t in the z direction. The conductor has charge carrier of charge q and drift velocity v_x when a current I_x flows in the positive x direction. The conductor is placed in a magnetic field perpendicular to the plane of the slab $\mathbf{B} = B_z \hat{z}$.

(a) When steady state is reached, there will be no net flow of charge in the y direction. Find the relation between E_y , B_z , and v_x .

(b) Find the resulting potential difference V_y (the **Hall voltage**) between the top and bottom of the slab, in terms of B_z , v_x , and the relevant dimensions of the slab.

(c) How do you determine the sign of the mobile charge carriers in a material? [Hint: n denotes the number of carriers per unit volume]



5. (7%,7%,6%) The magnetic field on the axis of a circular current loop is far from uniform. We can produce a more nearly uniform field by using two such circular loops a distance d apart. This arrangement is known as a **Helmholtz coil**.

(a) Find the total magnetic field $\mathbf{B}$ along the z -axis as a function of z .

(b) Show that $\partial B / \partial z$ is zero at the point midway between them.

(c) Determine d such that $\partial^2 B / \partial z^2 = 0$ at the midpoint, and find the resulting magnetic field at the center ($z=0$)

